Dynamics of vibrofluidized granular gases in periodic structures

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The behavior of a driven granular gas in a container consisting of M connected compartments is studied employing a microscopic kinetic model. After obtaining the governing equations for the occupation numbers and the granular temperatures of each compartment we consider the various dynamical regimes. The system displays interesting analogies with the ordering processes of phase separating mixtures quenched below their critical point. In particular, we show that below a certain value of the driving intensity the populations of the various compartments become unequal and the system forms clusters. Such a phenomenon is not instantaneous, but is characterized by a time scale τ which follows a Vogel-Vulcher exponential behavior. On the other hand, the reverse phenomenon which involves the ''evaporation'' of a cluster due to the driving force is also characterized by a second time scale which diverges at the limit of stability of the cluster.

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I. INTRODUCTION

One of the most ubiquitous phenomena in nature is the existence of matter in different phases. Water, for instance, can form either a solid, a liquid or a gas. When such a system is brought, by a sudden change of a control parameter, from a one phase equilibrium state to a point in the phase diagram where it can exist in two different phases, it will reach its new thermodynamic equilibrium only through a nonequilibrium process, called phase separation [1]. In the present paper, we stress that granular fluids possess a phenomenology which recalls closely that of standard matter. More specifically, we show that some processes observed during an ordinary condensation-evaporation process have a counterpart in vibrated granular fluids. In particular, the formation of dense patches surrounded by rarefied regions is similar to the phase separation dynamics associated with a liquid-gas transition [2].

The idea that granular fluids [3–5], i.e., large collections of inelastic particles fluidized by the action of an external driving force, under appropriate conditions may exhibit behaviors typical of ordinary fluids is contained in some recent papers. Sunthar and Kumaran reported the coexistence of different states in vibrofluidized granular beds [6]. Argentina *et al.*, instead, claimed that a vibrated gas of inelastic particles displays Van der Waals loops [7] in a pressure-density diagram.

Some years ago, Schlicting and Nordmeier in a seminal paper [8] considered an assembly of steel balls in a vertical container of height L partitioned into two connected sections, by a dividing wall of height $l<L$. They observed that when the container was vigorously shaken, the number of balls in the two sections was statistically identical, whereas the two populations were dramatically different for weak shaking. The reason for such a behavior is the competition between the particle diffusion induced by the shaking and the tendency to cluster resulting from the inelastic collisions. Various authors contributed with theoretical explanations of such a problem. These range from phenomenological flux models [9–12], to urn models [13–15], to kinetic theory approaches [16,17,19] . Moreover, the Twente group [12] presented new sets of experiments which have stimulated more interest in the problem.

We shall consider a granular gas subjected to a vigorous shaking and initially equipartitioned into several identical compartments, and show that it presents a phenomenology resembling that of spinodal decomposition [20]. At some instant $t=0$ the shaking intensity is decreased and the system evolves towards a new statistically steady state. The system has two possibilities: one is to persist in the homogeneous state and the second to cluster. The crossover between the two regimes occurs at a particular value of the driving intensity through the amplification of long-wavelength fluctuations. However, the analogy with spinodal decomposition is not complete, since in the late stage the order parameter does not saturate and the behavior of the granular system shows substantial differences with respect to familiar ordering systems.

The remainder of the paper is organized as follows. In Sec. II we specify the model, which is based on the assumption that the grains are described as inelastic hard disks subjected to a stochastic driving force. It is then possible to derive within the framework of the Boltzmann equation the governing law for the occupation number and the granular temperature of each compartment, the relevant variables in the problem. The governing equations represent an extension of those recently employed in Ref. [17] and validated by Monte Carlo simulation. In Sec. III we perform the stability analysis of the homogeneous asymptotic solutions of the governing equations and extract predictions about the crossover from the homogeneous regime to the clustered regime. In Sec. IV we illustrate the numerical results for one- and two-dimensional systems. Finally, in Sec. V we present our conclusions.

II. MODEL

We propose a simple extension of the model employed elsewhere [17] in order to study the steady state properties of a vibrofluidized granular gas to the case where the total vol-

ume available to the particles is divided into a series of identical compartments, and the grains can move from one to the other by jumping over a vertical wall. An assembly of $\mathcal{N}$ inelastic hard spheres moves in a d-dimensional domain partitioned into M identical regions of volume V separated by vertical obstacles. Each compartment contains N_i particles so that $\Sigma_{i=1}^{M} N_i = \mathcal{N}$.

When two particles collide their velocities after the collision, denoted with a prime, are obtained in terms of the (unprimed) precollisional velocities through the relation

$$\mathbf{v}_1' = \mathbf{v}_1 - \frac{1}{2}(1+\alpha)((\mathbf{v}_1 - \mathbf{v}_2)\cdot\hat{\boldsymbol{\sigma}})\hat{\boldsymbol{\sigma}}, \tag{1}$$

where $\hat{\boldsymbol{\sigma}}$ is the unit vector directed from particle 1 to particle 2, and α is the coefficient of restitution. A particle in the ith box, besides colliding inelastically with the remaining $(N_i - 1)$ within the same box, is subjected to the action of a white noise random force, which compensates the energy losses due to dissipative forces.

The dynamics of the kth particle between two successive collisions is based on a Langevin type equation of motion for each grain with a fluctuating random force accounting for the action of the external driving force:

$$\frac{d\mathbf{v}_k}{dt} = -\frac{1}{\tau_b}\mathbf{v}_k + \boldsymbol{\xi}_\mathbf{k}, \tag{2}$$

where $-\tau_b^{-1}\mathbf{v}_k$ is a viscous term and $\boldsymbol{\xi}_\mathbf{k}$ a Gaussian random acceleration, whose average is zero and variance satisfies a fluctuation-dissipation relation:

$$\langle \xi_{k\mu}(t)\xi_{m\nu}(t')\rangle = 2\frac{T_b}{m\tau_b}\delta_{km}\delta_{\mu\nu}\delta(t-t'), \tag{3}$$

where T_b is proportional to the intensity of the driving [21] and μ,ν denote vector components. The rate at which the kinetic energy is dissipated by collisions is proportional to $1-\alpha^2$.

Finally, the particles contained in compartment i can migrate into compartments $i\pm 1$ with a probability per unit time τ_s^{-1}, provided their kinetic energy exceeds the fixed threshold T_s, which is related to the gravitational energy necessary to overcome the vertical barrier.

Instead of considering the individual trajectories of the particles, one can study the single particle phase-space distribution function $f(\mathbf{r},\mathbf{v},t)$, which contains most of the relevant statistical information about the system. In order to obtain $f(\mathbf{r},\mathbf{v},t)$ one still has to solve the associated Boltzmann equation for $f(\mathbf{r},\mathbf{v},t)$ [18], which is not a simple task. However, a simpler description can be achieved by assuming that the relevant properties of the system are described in terms of the average particle population and of the average kinetic energy in each compartment. Therefore, we introduce the following coarse grained distributions:

$$f_i(\mathbf{v},t) = \frac{1}{V}\int_{V_i} d\mathbf{r} f(\mathbf{r},\mathbf{v},t),$$

where the integration domain is restricted to the volume of the ith compartment. Such an approximation clearly neglects the gradients which are present within each box.

Interestingly, also the coarse grained distributions $f_i(\mathbf{v},t)$ can be obtained by means of a Boltzmann-like kinetic approach. In fact, $f_i(\mathbf{v},t)$ evolves in time due to (a) the interaction with the heat bath, (b) the collisions between the particles belonging to the same box, and (c) particle diffusion from one compartment to the other. The effect due to (a) is represented by a Fokker-Planck term:

$$\frac{1}{\tau_b}\frac{\partial}{\partial\mathbf{v}_1}\left(\frac{T_b}{m}\frac{\partial}{\partial\mathbf{v}_1} + \mathbf{v}_1\right) f_i(\mathbf{v}_1,t), \tag{4}$$

whereas the effect of (b) is encapsulated in the collision term $I(f_i,f_i)$ (see Ref. [22]). Finally, we add a term taking into account the flux of particles between neighboring compartments. Since only fast particles contribute to the latter process such a term contains a Heaviside θ function. Collecting terms we obtain the following equation of motion for f_i

$$\begin{aligned}\partial_t f_i(\mathbf{v}_1,t) = I(f_i,f_i) &+ \frac{T_b}{m\tau_b}\left(\frac{\partial}{\partial\mathbf{v}_1}\right)^2 f_i(\mathbf{v}_1,t) \\ &+ \frac{1}{\tau_b}\frac{\partial}{\partial\mathbf{v}_1}[\mathbf{v}_1 f_i(\mathbf{v}_1,t)] - \frac{1}{\tau_s}\theta(|\mathbf{v}_1|-u_s) \\ &\times[2f_i(\mathbf{v}_1,t) - f_{i+1}(\mathbf{v}_1,t) - f_{i-1}(\mathbf{v}_1,t)].\end{aligned} \tag{5}$$

A more detailed treatment of such a transport equation can be found in Ref. [21] together with Ref. [17].

In order to characterize the macrostate of the system we only need the average number $N_i(t)$ of particles in compartment i at instant t, and the corresponding granular temperature T_i. These quantities are related to f_i by the equations

$$N_i(t) = \int d\mathbf{v} f_i(\mathbf{v},t), \tag{6}$$

where V_i is the volume of the same compartment, and

$$T_i(t) = \frac{1}{N_i(t)d}\int d\mathbf{v} m\mathbf{v}^2 f_i(\mathbf{v},t). \tag{7}$$

The problem can be reduced to a simple set of governing equations for the occupation numbers and the temperatures of the various compartment (see Ref. [17] for details) if one assumes a Gaussian shape for the distribution function f_i and $d=2$:

$$f_i(\mathbf{v},t) = \frac{N_i}{V}\frac{1}{2\pi T_i}\exp\left(-\frac{\mathbf{v}^2}{2mT_i}\right). \tag{8}$$

In the case of noncommunicating compartments $(\tau_s\to\infty)$, each containing $N^* = \mathcal{N}/M$ particles, the granular temperatures T_i assume the value T^*, determined by the solution of the equation

$$T^*\left[1 + \tau_b\sigma(1-\alpha^2)\frac{N^*}{2V}\sqrt{\frac{T^*}{m}}\right] = T_b. \tag{9}$$

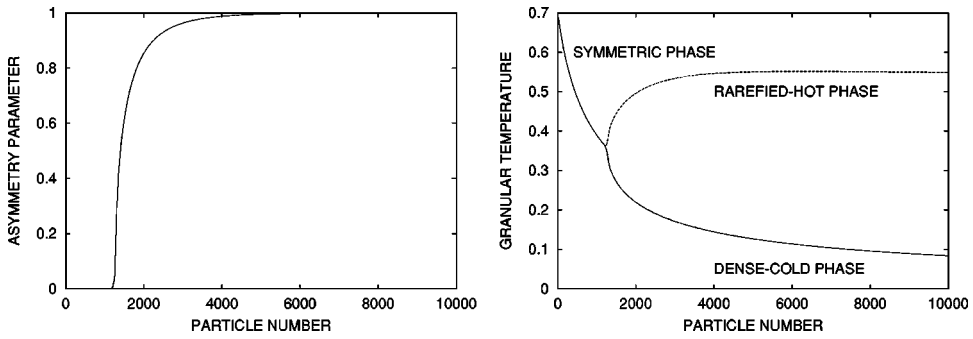

FIG. 1. Phase diagram relative to a system of two compartments: in (a) we show the asymmetry parameter $A=|N_L-N_R|/\mathcal{N}$ as a function of the total number of particles $\mathcal{N}$. In (b) granular temperatures of the two compartments are displayed as a function of $\mathcal{N}$. The heat-bath temperature is fixed at $T_b=0.7$, $T_s=1$, $\tau_b=2$, and $\tau_s=0.5$, while the compartment volume is $100\sigma^2$ and $\alpha=0.7$. The granular temperature is measured in nondimensional units.

In the general case, after simple manipulations, we obtain the following set of coupled equations:

$$\frac{dN_i(t)}{dt}=\frac{1}{\tau_s}[N_{i+1}e^{-T_s/T_{i+1}}+N_{i-1}e^{-T_s/T_{i-1}}-2N_ie^{-T_s/T_i}] \tag{10a}$$

$$\begin{aligned}N_i\frac{\mathrm{d}T_i(t)}{\mathrm{d}t}=&\frac{1}{\tau_s}[2(N_{i+1}T_{i+1}e^{-T_s/T_{i+1}}\\&+N_{i-1}T_{i-1}e^{-T_s/T_{i-1}}-2N_iT_ie^{-T_s/T_i})\\&+(N_{i+1}e^{-T_s/T_{i+1}}+N_{i-1}e^{-T_s/T_{i-1}}\\&-2N_ie^{-T_s/T_i})(2T_s-T_i)]-2\gamma\omega_iN_iT_i\\&+\frac{2}{\tau_b}N_i(T_b-T_i),\end{aligned} \tag{10b}$$

where the local dissipation rate [22] is

$$\gamma\omega_i=\sigma(1-\alpha^2)\frac{N_i}{2V}\sqrt{\frac{T_i}{m}}. \tag{11}$$

Let us notice that the ansatz (8) is only dictated by technical convenience and is not necessary, since it leads to a small set of coupled equations for N_i and T_i. In fact, one way to improve systematically the approximation is to assume f_i to be given by a Gaussian multiplied by a linear combination of orthogonal polynomials, the so-called Sonine polynomials. However, in this case one should consider extra variables besides N_i and T_i and the resulting set of equations would be of higher rank.

We turn now to consider the most relevant behaviors associated with the governing equations (10a) and (10b). Incidentally we notice that Eqs. (10a) and (10b) in the case of $M=2$ are identical to the corresponding equations of Ref. [17] after substituting τ_s with $2\tau_s$.

A large collection of grains initially distributed uniformly in an array of M identical compartments may remain homogeneous or may not according to the intensity of the driving to which it is subjected. The Twente group observed experimentally a homogeneous configuration under vigorous shaking, and clusters for weak shaking [12]. The crossover from the first to the second regime occurs when the ''thermally'' induced diffusion is not sufficient to prevent the spontaneous tendency of the grains to form clusters, due to the collisional cooling. Such an instability is the counterpart of the separation process which occurs in a system consisting of two compartments only. In such a case the left-right symmetry, i.e., the difference between the left population N_L and the right population N_R is spontaneously broken below a certain critical temperature. Figure 1(a) illustrates the corresponding behavior of the asymmetry parameter $A=|N_L-N_R|/\mathcal{N}$ versus the total number of particles for a fixed value of $T_b=0.7$, whereas in Fig. 1(b) we display the variation of the granular temperatures within the two compartments versus the total number of particles. The curve bifurcates at a critical value of the total number of particles identifying a temperature T_c below which the left-right symmetry is broken. We shall discuss how such a mechanism manifests in the case of many compartments and gives rise to a phenomenology similar to that of the spinodal decomposition [20]. In general the space of dimensionless control parameter is large since the system properties are functions of α, M, $\mathcal{N}$, V/σ^2, T_b/T_s, τ_b/τ_s, and $\gamma\tau_s$. In the following we shall measure temperatures in units of T_s, areas in units of σ^2, and time in units of $\tau_b/2$.

III. STABILITY ANALYSIS OF THE HOMOGENEOUS STATE

In the case of M identical compartments with cyclic boundary conditions, the choice $N_i=N^*$ and $T_i=T^*$, where T^* and N^* are related by Eq. (9), represents a uniform solution of Eqs. (10), for all values of the control parameters. We observe that T^* is the granular temperature of a system of $\mathcal{N}$ particles, equally distributed into M compartments of volume V and subjected to a heat bath T_b.

On the other hand, it turns out that such a uniform solution is stable only at high temperature, where a diffusive mechanism tends to restore any small perturbation about the homogeneous state. On the contrary, the uniform state below a certain temperature turns out to be unstable with respect to spontaneous fluctuations, due to the clustering mechanism induced by inelasticity. As shown in Fig. 1(b), the granular temperature in the case of a system with two compartments takes on two different values when the total population ex-

ceeds a threshold value. In the case of many connected compartments a related phenomenon occurs. In order to illustrate it, we introduce a small amplitude sinusoidal perturbation $T_l = T^* + \delta T_k \exp(ikl)$, and $N_l = N^* + \delta N_k \exp(ikl)$, where $k = 2\pi n/M$, with $n = 1, \ldots, M-1$ and $l = 1, \ldots, M$ denotes the compartment.

Expanding linearly Eqs. (10a) and (10b) about the symmetric fixed point T^*, N^* one finds the following result:

$$\delta \dot{N}_k = -\frac{e^{-T_s/T^*}}{\tau_s} 2[1-\cos(k)]\left[\delta N_k + \frac{N^* T_s}{(T^*)^2}\delta T_k\right] \tag{12a}$$

$$\delta \dot{T}_k = -\frac{e^{-T_s/T^*}}{\tau_s}\left\{\left[2+\frac{T_s}{T^*}+2\left(\frac{T_s}{T^*}\right)^2\right][1-\cos(k)] + \left(3\gamma\omega^* + \frac{2}{\tau_b}\right)\right\}\delta T_k - \frac{2}{N^*} \times \left[\frac{e^{-T_s/T^*}}{\tau_s}(T^*+2T_s)[1-\cos(k)] + \gamma\omega^* T^*\right]\delta N_k . \tag{12b}$$

The associated eigenvalues $\lambda_n(k)$ and $\lambda_T(k)$ of the dynamical matrix of coefficients correspond to the two relaxation modes of the system. The larger eigenvalue $\lambda_n(k)$ vanishes quadratically when $k \to 0$, reflecting the conservation of the global number of particles. However, due to the coupling between the density and the thermal fluctuations, for finite k, $\lambda_n(k)$ presents a variety of behaviors, associated with different physical phenomena.

In order to classify these behaviors, we consider the following expansion valid for small k values:

$$\lambda_n(k) = a_2 k^2 + a_4 k^4. \tag{13}$$

Above T_c, a_2 is negative, the eigenvalue $\lambda_n(k)$ describes a diffusion process by which a local density fluctuation is readsorbed. In other words, the particle fluxes caused by the coupling to the external driving are sufficient to restore homogeneity.

Below a certain characteristic temperature T_c, a fluctuation which increases locally the population is amplified. The local granular temperature drops due to the increased collision rate, since [from Eq. (9)] $\omega_l \propto N_l T_l^{1/2} \propto N_l^{2/3}$ and $T_l \propto N_l^{-2/3}$. Thus the particles arriving from the other compartments remain trapped, causing a further reduction of the local temperature. This phenomenon is described by the formula

$$a_2 = \frac{e^{-T_s/T^*}}{\tau_s}\left[\frac{T_c}{T^*} - 1\right], \tag{14}$$

i.e., a_2 becomes positive below T_c which is given by

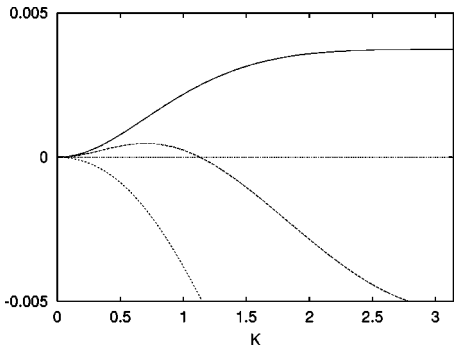

FIG. 2. Variation of the eigenvalue $\lambda_n(k)$ with respect to the wave vector k for different choices of the heat-bath temperature $T_b = 0.28$ (continuous line), $T_b = 0.38$ (dashed line), and $T_b = 0.45$ (dotted line). The data refer to a system consisting of 100 compartments, each initially filled with 180 particles. The compartment volume is $100\sigma^2$ and $\alpha = 0.7$. The threshold temperature is $T_s = 1$, $\tau_b = 2$, $\tau_s = 0.5$. Notice that the mode becomes unstable, i.e., $\lambda_n(k)$ becomes positive, below the critical temperature.

$$T_c = \frac{T_s}{\frac{3}{2} + \frac{1}{\tau_b \gamma\omega^*}} \tag{15}$$

with $\gamma\omega^* = \sigma(1-\alpha^2)(N^*/2V)\sqrt{T^*/m}$. Contrary to the previous diffusive case, particles tend to cluster.

According to the sign of a_4 (for $a_2 > 0$) the initial regime is different. In fact, if $a_4 < 0$, $\lambda(k)$ may display a maximum at a finite wave vector $k_m < 2\pi$ ($T_b = 0.38$, in Fig. 2), whereas for $a_4 > 0$, $\lambda_n(k)$ attains its maximum only in correspondence of the largest wave vector for $T_b = 0.28$.

Correspondingly, in the second case the growth process initially resembles the early stage of the spinodal decomposition process, during which the homogeneous state is unstable with respect to long-wavelength fluctuations. These are exponentially amplified, whereas short scale fluctuations decay.

Physically speaking, a local density increment induces a decrement in the granular temperature. If the temperature of the bath is sufficiently low or the number of particles sufficiently large this leads to an instability, i.e., more particles will flow toward the region where the temperature is lower. Diffusion will not be able to compensate such a tendency.

Finally, there exists a second collective mode $\lambda_T(k)$, which essentially describes how temperature fluctuations decay. It is always negative, due to the presence of dissipation caused by friction and collisions, and given by

$$\lambda_T(k) = -\left(3\gamma\omega^* + \frac{2}{\tau_b}\right) - ck^2. \tag{16}$$

IV. NUMERICAL RESULTS

In the present section we shall study numerically the properties of the model. We have integrated Eqs. (10) with $\alpha = 0.7$, $V = 100\sigma^2$, $T_s = 1$, $\tau_b = 2$, $\tau_s = 0.5$. We have chosen σ as unit length, T_s as energy unit, and $2\tau_s$ as time unit. The

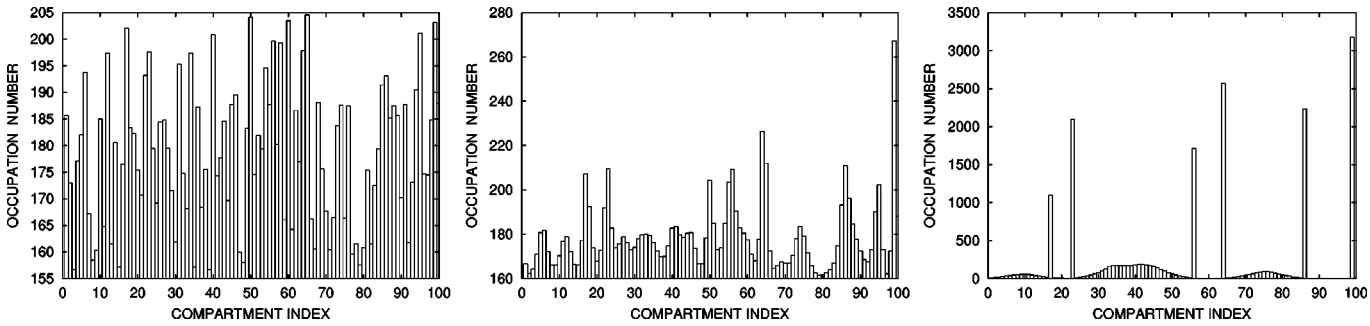

FIG. 3. Evolution of the occupation profile relative to a system constituted by 100 compartments and 180 particles initially placed in each compartment. The temperature is chosen below the critical temperature and is $T_b=0.38$, while the remaining parameters are the same as in Fig. 2. The three snapshots from left to right refer to times $t_1=500$, $t_2=1500$, and $t_3=3000$ in units of τ_b.

value of T_b and of $\mathcal{N}$ and M varied from case to case, together with the initial conditions.

The stability analysis discussed so far describes only the linear, early stage of the evolution. In order to explore the behavior in the late nonlinear regime we have solved numerically the governing equations (10). We considered a one-dimensional array of compartments, initially equally populated, at the same granular temperature, i.e., $N_i=N^*$ and $T_i=T^*$, and added a small random perturbation. According to the previous linear analysis, two different behaviors can be observed as the control parameters, such as the heatbath temperature and the average density vary. At fixed N, for large values of T_b, such that $T_g>T_c$, the initial perturbation is readsorbed diffusively, while at small values of $T_b(T_g<T_c)$ the perturbation is exponentially amplified. In the latter case, the collisional cooling determines a decrease of the local temperature in correspondence of the regions more populated and clustering begins. Some compartments, randomly selected by the dynamics, act as germs for the nucleation process illustrated in Fig. 3. After the initial regime few compartments grow at the expense of the remaining which become empty. The distance between highly populated compartments increases, since they compete for particles. However, unlike the late stage spinodal decomposition process [20], we observe that the domains do not grow in size, but in height. This feature can be understood because no saturation mechanism is present in the model, so that the occupation number in a single compartment can become of the order of the total population $\mathcal{N}$. The growth process occurs by diffusion against the density gradient, since particles move from low populated regions toward highly populated regions. The evolution of the width $W(t)$ (see Fig. 4),

$$W(t)=\sqrt{\frac{1}{M}\sum_{i=1}^{M}[N_i^2(t)-(N^*)^2]}, \tag{17}$$

illustrates quantitatively how the process occurs. During the initial stage $W(t)$ remains much smaller than N^* and only after a characteristic time τ it begins to rise steeply. In Fig. 4 we show $W(t)$ versus time and the parametric plot $W(t)$ against the average value of the granular temperature. We notice that the latter plot recalls an order parameter versus temperature plot in a system undergoing a phase transition.

A second quantitative measure of the clustering phenomenon is represented by the following statistical indicator:

$$h=-\sum_{i}^{M}\frac{N_i}{\mathcal{N}}\ln\left(\frac{N_i}{\mathcal{N}}\right). \tag{18}$$

The ''entropy'' h is non-negative, vanishes when all particles are confined in a single compartment, and takes on its maximum value $\ln(M)$ when all compartments are identically populated. Thus $f=\exp(h)$ represents a measure of the number of occupied compartments. In Fig. 5 we display the evolution of the average granular temperature and of f in various situations. Above T_c the indicator f relaxes toward M, whereas in the low temperature region, due to clustering, f decreases toward a plateau value $P<M$. Interestingly, such a relaxation time τ increases as the system approaches the tem-

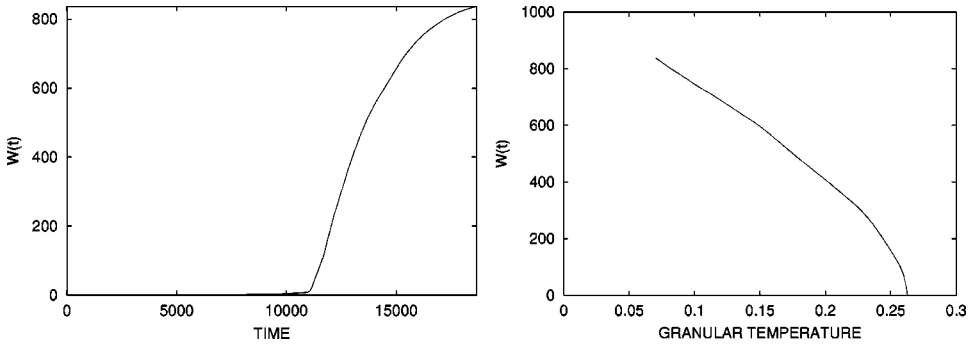

FIG. 4. Growth of the profile width relative to a system constituted by 100 compartments and 180 particles initially placed in each compartment. The heat-bath temperature is $T_b=0.38$ and the remaining parameters are the same as in Fig. 2. The right figure represents the parametric plot $W(t)$ vs the average granular temperature for the same system.

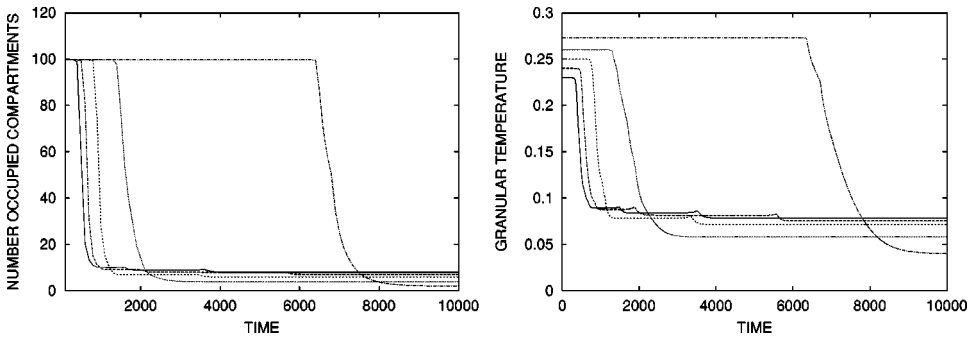

FIG. 5. Time evolution of the average number of occupied compartments, $f(t)$, relative to a system with $M=100$ compartments and 180 particles per compartment at $t=0$. The curves refer from right to left to granular temperatures $T_g=0.273$, 0.26, 0.25, 0.24, 0.23. The remaining parameters are those of Fig. 2. In (b) we display the variation of the average granular temperature for the same system (b).

perature T_c from below. The temperature dependence of τ close to T_c, displayed in Fig. 6, is consistent with the Vogel-Fulcher law:

$$\tau_{vf}=A\exp[\Delta/(T-T_o)]. \tag{19}$$

The dependence of the characteristic time τ_{vf} on the temperature is a direct consequence of Eqs. (13) and (14). We also remark that the plateau value of *f*, reached by the system for $t>\tau_{vf}$, increases as T_b decreases, indicating that the system remains trapped in some metastable configurations.

We now turn our attention to a different process obtained by considering the evolution of an initial configuration, in which all the particles are located inside a single compartment at $t=0$. According to the level of T_b one can observe two different processes: a) for large T_b the occupation number in the central cell decays toward the fully symmetric state $N_i=\mathcal{N}/M$ and $\lim_{t\to\infty}f(t)=M$; b) for small T_b the occupation of the compartment remains constant.

In Fig. 7 we display the variation in time of the population in the compartment, where it was initially placed, for various values of the heat-bath temperature. We observe that $N(t)$ decreases more and more slowly as the transition temperature is approached from above.

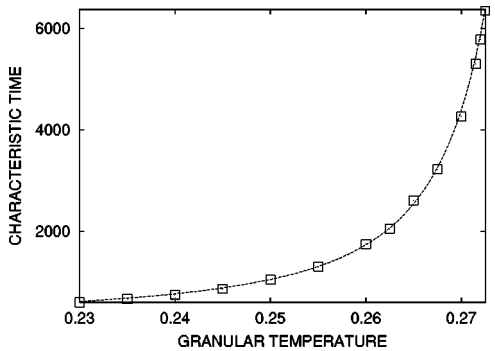

FIG. 6. Characteristic time associated with the position of the kink, shown in Fig. 5(a), vs average granular temperature. The system is a linear array with $M=100$ compartments. The remaining control parameters are the same as in Fig. 2. The behavior is not Ahrrenius-like. The data can be fitted by a Vogel-Fulcher law $\tau_{vf}=A\exp[\Delta/(T_g-T_o)]$, where the fitting parameters are $A=200$, $\Delta=0.072$, and $T_o=0.293$.

Figure 8 shows the temperature dependence of the average time τ necessary to wash out the initial single cluster configuration. We observe that $\tau=C/(T_b-T_p)^{3/2}$ diverges at the crossover temperature T_p. In particular the plot of the occupation number versus time indicates the growth of a plateau when $T_b\to T_p$. The smaller the temperature deviation from the limit of stability T_p the longer the plateau.

In Fig. 9 we display the transition temperature at which a cluster of $N=1200$ particles placed in a cell at $t=0$ ''explodes'' as a function of the number of boxes, *M*. Such an effect has been observed experimentally by van der Weele *et al.* [10] and explained in terms of their flux model.

Finally, we consider the inverse process, by which a cluster instead of ''evaporating'' grows at the expenses of its neighbors. Such a study also provides some information about the interfacial properties of the model. We prepare the system in an initial configuration where the population N_c of a single compartment is much larger than that of the remaining compartments. If the profile were uniform with $N_i=N_\infty=180$ and $T_b=0.7$ the state would be linearly stable with respect to perturbations. However, if we place $N_c=1800$ particles in one of the compartments and keep the remaining populations at $N_i=180$ we observe the following scenario. The spike grows in height, while the compartments closer to it slowly become empty. As shown in Fig. 10 the population profile develops a gradient due to the flux of particles from the ''bulk'' to the ''tower.'' Its shape is similar to the depletion layer associated with the growth of a liquid droplet in a

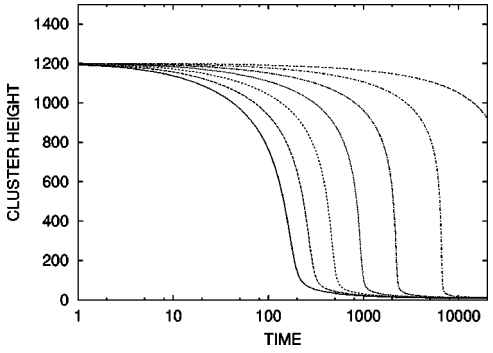

FIG. 7. Variation of the height of a cluster, initially containing 1200 particles, as a function of time for temperatures $T_b=0.30$, 0.35, 0.40, 0.45, 0.50, 0.55, and 0.60 from top to bottom.

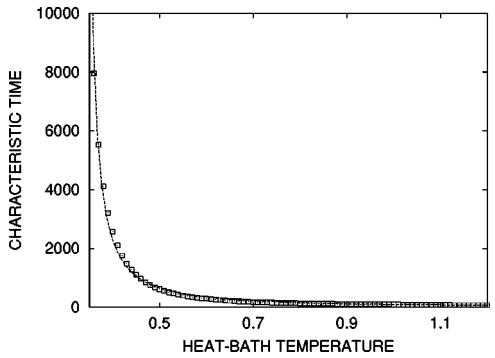

FIG. 8. Characteristic time associated with the explosion of a cluster of 1200 particles located at $t=0$ in a single compartment vs the heath-bath temperature. The system consists of a linear array of 100 compartments, while the remaining parameters are the same as in Fig. 2

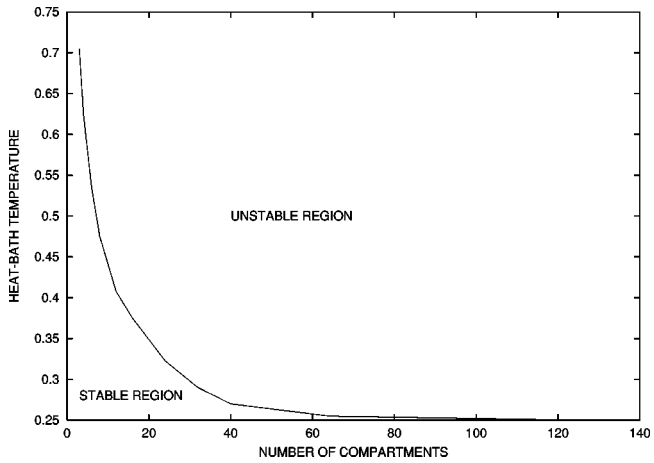

FIG. 9. Stability of an isolated cluster as a function of the number of compartments. We show the phase boundary between the stable and the unstable regimes relative to a cluster of $N=1200$ particles initially placed in a single compartment. The vertical axis represents the heat-bath temperature at which the transition takes place, while the horizontal axis represents the number of compartments.

''sea'' of oversaturated gas [23]. In addition, we observed that since the flux terms in Eq. (12a) are very small the local values of the granular temperature and of the occupation number are related by Eq. (9). The profile varies almost linearly indicating that the process can be assimilated to a diffusion controlled interfacial growth.

It is worthwhile to comment that the model discussed above presents a shortcoming, since the local density can grow unbounded until all the particles in the system occupy the same well. In fact, the approximation scheme does not treat adequately the mutual repulsion between the particles at high densities. Particles are allowed to pile up and reach arbitrarily large packing fractions. In order to eliminate such a shortcoming of the model, it is possible to introduce phenomenologically a new ingredient into the theory. The model was modified introducing an occupation number dependence in the characteristic time τ_s:

$$\tau_s(N_i)=\tau_o(1-N_i/M)^{1/2}, \tag{20}$$

that is, the probability of escaping from a compartment be-

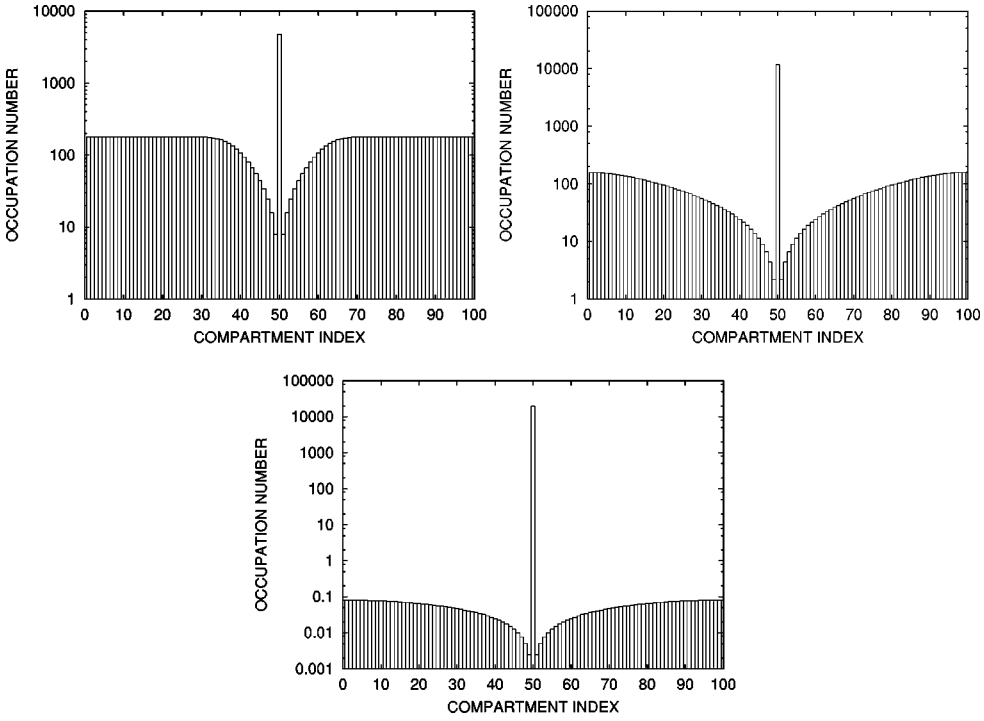

FIG. 10. Evolution of the depletion layer around a growing cluster at times $t=500$, $t=2500$, and $t=20\,000$ for $M=100$. The initial configuration is $N_i=180$, $N_{50}=1800$, and $T_b=0.7$. The system starts with a central peak and a supersaturated ''bulk.'' As the system evolves we observe the increasing thickness of the depletion layer. Matter accumulates in the central compartment.

comes larger when it is occupied by a large number of grains. This correction is based on the results of microscopic studies [24]. The form above has been chosen in order to preserve the form of the equations as simple as possible and by phenomenological requirements. We observed that during the early stages of the process, where the linear analysis of Sec. III applies, there are no appreciable changes to the situation described in the previous sections. On the other hand, the late stage presents a different scenario. When the compartments saturate the occupation number and the granular temperature become locally stationary and the growth proceeds in the neighboring compartments and so on.

V. CONCLUSIONS

To summarize, we introduced a model for compartmentalized driven granular gases and studied it using the methods of kinetic theory. We have found a rather rich ''phase'' behavior and the emergence of different qualitative properties as the number of particles becomes sufficiently large. We have pointed out that the system undergoes a long-wavelength instability and orders in a fashion similar to the process which occurs during the spinodal decomposition in fluid mixtures. However, the late stage of the process is radically different, because the granular gas does not possess a surface tension mechanism which restores homogeneity. Thus the usual competition between bulk and surface free energy cost which determines the growth of larger and larger domains is not at work here.

The present approach, in spite of its simplicity, has the advantage with respect to the so-called flux models of relating the microscopic parameters to the macroscopic observables in a natural fashion. It can be extended to treat granular mixtures, where the granular temperature of each component must be treated as an independent variable to be determined self-consistently [25,26]. In addition, the approach can be improved by including non-Gaussian corrections to the distribution function or by solving numerically the Boltzmann equation by the Direct Monte Carlo simulation method.

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